The Impact of Information Technology Architecture on Supply Chain Performance

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Abstract

Supply chain oscillations satisfy a sound wave-like dispersion relation when a company responds only to the status of the companies immediately above and below it in the chain. However, when information exchange makes it possible for the company to respond to the status of all the companies in the chain, the dispersion relation changes from that of a sound wave to that of a plasma oscillation. The plasma oscillation exhibits Landau damping, and thus the information exchange leads to beneficial suppression of the oscillations.

1. Introduction

Supply chains are notorious for exhibiting oscillations in inventories that are both disruptive and costly in resources. Business schools have for several years exposed their students to the phenomena through the widely used simulation game created by J. D. Sterman and his colleagues at MIT [Sterman and Fiddaman (1993)].

Sterman and Fiddaman conjectured that the oscillations were due in part to the lack of information exchange between the companies in the change. This lack of information exchange prevents controlled responses and leads to over reaction to perturbations from the steady state.

Recently, a simple model was developed in which each company in the supply chain responded only to the status of the companies immediately above and below it in the chain [Dozier and Chang (2005a)]. The model displayed the types of supply chain oscillations observed in both the simulations and in actual practice. The oscillations satisfied the same type of dispersion relation as acoustic waves in a solid. In a follow-on paper, a crude continuum flowing fluid model of the supply chain was introduced: in the flowing fluid model, the resulting supply chain oscillations were found to satisfy the same dispersion relation as sound waves [Dozier and Chang (2005b)].

The purpose of this paper is to explore what happens if information exchange occurs between all the companies in the supply chain. This enables each company to respond not just to the inventory status of the layers immediately below and above it in the chain, but to the inventory status of all the companies in the chain. It will be shown that the oscillations change their character and become more like plasma oscillations than sound waves. The associated Landau damping of the oscillations suggests that information exchange leads to beneficial suppression of the oscillations.

Section 2 presents a more realistic model of local information exchange a supply chain than our earlier treatments, in order to provide an easy comparison with the treatment of universal information exchange.

Section 3 derives the oscillation dispersion relation for a supply chain in which there is information exchange with all the companies in the chain.

Section 4 discusses the results and their implications.

2. Supply chain with local exchange of information

In Dozier and Chang (2005b), the supply chain was treated in the continuum limit where instead of designating each level in the chain by a discrete label n, the position in a chain was designated by a continuum variable x. Flow of production through each position *x* in the chain was characterized by a velocity variable v. A long supply chain was treated in which end effects were ignored.

We begin by introducing a function of position, production flow rate velocity, and time, f(x,v,t)dxdv that denotes a flow in the number of production units in the intervals dx and dv at a given x and v at the time t. This distribution function can be expressed as a conservation equation in the phase space of x and v:

$$\partial f/\partial t + \partial[f dx/dt]/\partial x + \partial[f dv/dt]/\partial v = 0 \qquad [1]$$

This equation simply states that the change of fdxdv is due only to the divergence of the flow into dxdv. This implies that the flow into a volume element dxdv may not the same as the flow out.

By introducing a force F that influences the velocity of the production rate v , this equation can be rewritten

$$\partial f/\partial t + \partial[fv]/\partial x + \partial[fF]/\partial v = 0 \qquad [2]$$

Since position x and velocity of the production rate v are independent variables,

$$\partial v/\partial x = 0 \qquad [3]$$

If. moreover, the force F does not depend on v,

$$\partial F/\partial v = 0 \qquad [4]$$

then eqs [2]-[4] yield

$$\partial f/\partial t + v\partial f/\partial x + F\partial f/\partial v = 0 \qquad [5]$$

This has the familiar form of the Vlasov equation for collisionless plasmas [Spitzer (1956)].

Now assume that F at the position x is determined only by the level of the inventories of the production units immediately above and below x in the chain. Assume that the fractional change in the time rate of change of velocity (1/v)dv/dt is proportional to the fractional change in the gradient of the density *N*(x,t):

$$(1/v)dv/dt \propto - (1/N)dN/dx \qquad [6]$$

where

$$N(x,t) = \int dv f(x,v,t) \qquad [7]$$

and where the negative sign is explained below.

For local information exchange with the levels immediately above and below the level of interest, the change in the density is observed over only dx =2*l*, where *l* is between levels in the supply chain   Thus, we can further write

$$(1/v)dv/dt \propto - (21/N)dN/dx \qquad [8]$$

The rationale for this expression is that when the inventory of the level below the level of interest is less than normal, the production rate (v) will be diminished because of the smaller number of production units being introduced to that level. At the same time, when the inventory of the level above the level of interest is larger than normal, the production rate will also be diminished because the upper level will demand less input so that it can "catch up" in its production through-put. Both effects give production rate changes proportional to the gradient of *N*. It is reasonable also that the fractional changes are related rather than the changes themselves, since deviations are always made from the inventories at hand.

A time scale for the response is missing from eq. [8]. We know that a firm must make decisions on how to react to order flows into the firm. Assume that the time scale of response $\tau_{response}$ is given by

$$\tau_{response} = (1/\xi)\tau_{processing} \qquad [9]$$

where $\tau_{processing}$ is the processing time for a unit as it passes through the firm, and for simplification we are assuming ξ is a constant. Most likely, ξ will be less than unity, corresponding to response times being longer than processing times.

Thus, eqs. [6] - [9] lead to

$$(1/v)dv/dt \quad - (2\xi 1/\tau_{processing}\ n)dn/dx \qquad [10]$$

Since by definition production rate velocity

$$v = l/\tau_{processing} \qquad [11],$$

this gives finally

$$F = dv/dt = -\ 2\xi v^2(1/N)dN/dx \quad [12]$$

Insertion of this expression into eq. [5] then yields

$$\partial f/\partial t + v\partial f/\partial x - 2\xi v^2(1/N)(dN/dx)\ \partial f/\partial v = 0 \quad [13]$$

In the steady state, the equation is satisfied by

$$f(x,v,t) = f_0(v) \quad [14]$$

i.e. by a distribution function that is independent of position and time. For a smoothly operating supply chain, $f_0(v)$ will be centered about some flow velocity V_0.

Now suppose there is a (normal mode) perturbation of the form $\exp[i(\omega t - kx)]$, i.e.

$$f(x,v,t) = f_0(v) + f_1(v)\exp[-i(\omega t - kx)] \quad [15]$$

On linearizing eq. [13] with this f(x,v,t), we find that $f_1(v)$ satisfies

$$-i(\omega\text{-}kv)f_1 - 2\xi v^2(1/N_0)(dN_1/dx)\ \partial f_0/\partial v = 0 \quad [16a]$$

i.e.

$$-i(\omega\text{-}kv)f_1 - ik\ 2\xi v^2(1/N_0)N_1\partial f_0/\partial v = 0 \quad [16b]$$

Solving for f_1:

$$f_1 = -2\xi k(1/N_0)\int dv' f_1(v')\ v^2\partial f_0/\partial v(\omega\text{-}kv)^{-1} \quad [17]$$

On integrating this equation with respect to v, we get the dispersion relation relating ω and k:

$$1+ 2\xi k\ (1/N_0)\int dv v^2\partial f_0/\partial v(\omega\text{-}kv)^{-1} = 0 \quad [18]$$

This equation contains a singularity at ω=kv. Following the Landau prescription [Landau (1946); Stix (1962)]

$$\int dv v^2\partial f_0/\partial v(\omega\text{-}kv)^{-1} = PP\int dv v^2\partial f_0/\partial v(\omega\text{-}kv)^{-1} - i\pi(\omega/k)^2(1/k)\partial f_0(\omega/k)/\partial v \quad [19]$$

where PP denotes the principal part of the integral.

To evaluate the principal part, assume that for most v, ω>>kv. Then approximately

$$PP\int dv\, v^2 \partial f_0/\partial v(\omega-kv)^{-1} \approx \int dv\, v^2 \partial f_0/\partial v(1/\omega) \qquad [20a]$$

or, on integrating this by parts, we find

$$PP\int dv\, v^2 \partial f_0/\partial v(\omega-kv)^{-1} \approx - 2n_0V_0 \qquad [20b]$$

since f_0 is peaked about the equilibrium flow velocity V_0

This gives the sound-wave-like dispersion relation

$$\omega \approx 4\xi kV_0 \qquad [21]$$

Addition to this of the small contribution from the imaginary part yields

$$\omega = 4\xi kV_0 + \omega(1/N_0)i\pi(\omega/k)^2\partial f_0(\omega/k)/\partial v \qquad [22]$$

or, on using the approximate relationship of eq. [21] for the ω's in the second term on the RHS

$$\omega = 4\xi kV_0\,[1+ (1/N_0)i\pi(4\xi V_0)^2\partial f_0(4\xi V_0)/\partial v] \qquad [23]$$

For the fast response times made possible by first order rapid information exchange, $\xi = O(1)$. Thus, with $f_0(v)$ peaked around V_0, $\partial f_0(4\xi V_0)/\partial v < 0$.

Accordingly, the imaginary part of ω is less than zero, and this corresponds to a damping of the normal mode oscillation. Since $4\xi V_0 >> V_0$ (where the distribution is peaked), the derivative will be small, however, and the damping will be correspondingly small.

2. Supply chain with universal exchange of information

Consider next what happens if the exchange of information is not just local. In this case, the force F in eq. [5] is not just dependent on the levels above and below the level of interest, but on the f(x,v,t) at all x.
Let us assume that the effect of f(x,v,t) on a level is independent of what the position of x. This can be described by introducing a potential function Φ that depends on f(x,v,t,) by the relation

$$\partial^2\Phi/\partial x^2 = - [C/N_0]\int dv\, f(x,v,t) \qquad [24]$$

from which the force F is obtained as

$$F = - \partial\Phi/\partial x \qquad [25]$$

The constant C can be determined by having F reduce approximately to the expression of eq. [12] when f(x,v,t) is non zero only for the levels immediately above and below the level x_0 of interest in the chain. For that case, take

$$N(x+l) = N(x_0) + dN/dx\ l \qquad [26]$$

and

$$N(x- l) = N(x_0) - dN/dx\ l \qquad [27]$$

and zero elsewhere. Then

$$F = - \partial\Phi/\partial x = - [C/N_0](dN/dx)\ 2l^2 \qquad [28]$$

On comparing this with the F of eq. [12], $F = - 2\xi v^2(1/N)dN/dx$, we find (since the distribution function is peaked at V_0) that we can write

$$C = \xi V_0^2 / l^2 \qquad [29]$$

Accordingly,

$$\partial^2\Phi/\partial x^2 = - [\xi V_0^2 / N_0 l^2]\int dv\ f(x,v,t) \qquad [30]$$

With these relations, F from the same value of f(x,v,t) at all x above the level of interest is the same, and F from the same value of f(x,v,t) at all x below the level of interest is the same but of opposite sign.

This is the desired generalization from local information exchange to universal information exchange.

It is interesting to see what change this makes in the dispersion relation. Equation [5] now becomes

$$\partial f/\partial t + v\partial f/\partial x - \partial\Phi/\partial x\ \partial f/\partial v = 0 \qquad [31]$$

and again the dispersion relation can be obtained from this equation by introducing a perturbation of the form of eq. [15] and assuming that Φ is of first order in the perturbation. This gives

$$-i(\omega-kv)f_1 = ik\Phi_1\partial f_0 /\partial v \qquad [32]$$

i.e.,

$$f_1 = -k\Phi_1\partial f_0 /\partial v\ (\omega-kv)^{-1} \qquad [33]$$

Since eq. [30] implies

$$\Phi_1 = (1/k^2)\,[\xi V_0^2 / N_0 l^2] \int dv\, f_1(v) \qquad [34]$$

we get on integrating eq. [33] over v:

$$1 + (1/k)\,[\xi V_0^2 / N_0 l^2] \int dv\, \partial f_0 / \partial v\, (\omega - kv)^{-1} = 0 \qquad [35]$$

Once again a singularity appears in the integral, so we write

$$\int dv\, \partial f_0 / \partial v\, (\omega - kv)^{-1} = PP\int dv\, \partial f_0 / \partial v\, (\omega - kv)^{-1} - i\pi (1/k) \partial f_0(\omega/k) / \partial v \qquad [36]$$

Evaluate the principal part by moving into the frame of reference moving at V_0, and in that frame assume that $kv/\omega << 1$:

$$PP\int dv\, \partial f_0 / \partial v\, (\omega - kv)^{-1} \approx \int dv\, \partial f_0 / \partial v\, (1/\omega)[1 + (kv/\omega)]$$

$$= -kN_0/\omega^2 \qquad [37]$$

Moving back into the frame where the supply chain is stationary,

$$PP\int dv\, \partial f_0 / \partial v\, (\omega - kv)^{-1} \approx -kN_0/(\omega - kV_0)^2 \qquad [38]$$

This gives the approximate dispersion relation

$$1 - (1/k)\,[\xi V_0^2 / N_0 l^2]\, kN_0/(\omega - kV_0)^2 \approx 0 \qquad [39]$$

i.e.

$$\omega = kV_0 + \xi^{1/2} V_0/l \quad \text{or } \omega = kV_0 - \xi^{1/2} V_0/l \qquad [40]$$

To assure that $\omega > 0$ as $k \to 0$, we shall discard the minus solution as spurious.

Now add the small imaginary part to the integral:

$$1 + (1/k)\,[\xi V_0^2 / N_0 l^2][-kN_0/(\omega - kV_0)^2 - i\pi(1/k)\partial f_0(\omega/k) / \partial v] = 0 \qquad [41]$$

On iteration, this yields

$$\omega \approx kV_0 + \xi^{1/2}(V_0/l)\,[1 + i\,\{\pi \xi V_0^2/(2k^2 l^2 N_0)\}\partial f_0/\partial v\,] \qquad [42]$$

where $\partial f_0/\partial v$ is evaluated at

$$v = \omega/k \approx V_0 + (\xi^{1/2} V_0/kl) \qquad [43]$$

Since for velocities greater than V_0, $\partial f_0/\partial v < 0$, we see that the oscillation is damped.

Universal information exchange has resulted both in changing the form of the supply chain oscillation and in suppression of the resulting oscillation.

4. Discussion

The purpose of this paper has been to introduce a simple flow model for comparing the impacts of local information exchange *to* universal information exchange in a supply chain. The local information exchange has been described by a term that describes the interaction of a company with those immediately above and below it in the supply chain. The universal information exchange has been described by introducing a potential that satisfies a Laplace equation. This potential corresponds to each company above the company at the location of interest contributing equally to that company's actions, and to each company below the company at the location of interest contributing equally but oppositely to that company's actions.

It has been demonstrated that for local information exchange, the dispersion relation that describes the relation between frequency of oscillation and the wave number of the oscillation, resembles that for a sound wave in a flowing fluid, i.e. the wave velocity of the perturbations is proportional to the wave number, and is greater than the production flow velocity. These waves are damped, but the damping can be small because the phase velocity is so much larger than the flow velocity.

It has also been shown that for universal information exchange, the dispersion relation resembles that for a plasma oscillation. Instead of the frequency being proportional to the wave number, as in the local information exchange case, the frequency now contains a component which is independent of wave number. The plasma-like oscillations for the universal information exchange case are always damped. As the wave number k becomes large, the damping (which is proportional to $\partial f_0 (\omega/k) / \partial v$) can become large as the phase velocity approaches closer to the flow velocity V_0.

Accordingly, the simple flow model of supply chains has demonstrated that universal information exchange both changes the character of the supply chain oscillations and suppresses the oscillations. This supports Sterman and Fiddaman's conjecture that IT will have beneficial effects on supply chains.

The conclusions of this paper have been based on a rather crude flow model of supply chains and on some rather approximate treatments of the associated equations. Nevertheless, it is hoped that the model has helped develop an intuitive understanding of the different effects.

Future work will create numerical simulations hat compares the undampened oscillations incurred by serial communication to the predicted dampened oscillations of grid communication.

References

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