**ARCHIVES PAPER**

# Massachusetts Institute of Technology, School of Industrial Management

*Note to the Faculty Research Seminar. From: Jay W. Forrester. November 5, 1956*

*This is arguably the definitive archives paper in the history of system dynamics; indeed it could be described as representing the genesis of the field. Written two years before the commonly cited 1958 Harvard Business Review (**36**:4) paper, this first ever of the MIT 'D-memos' (D-memo zero) sets out the disadvantages and difficulties associated with the style of (economic) modelling as had gone on before and signposts the future by portraying the distinguishing features of the methodology we are now so familiar with.*

*The paper has hardly dated over these past 47 years and repays a read for those who consider themselves as competent in the field. But the novice will have much more to gain in understanding the rationale and world-view of the practitioner of system dynamics.*

*The original, on 27 pages of closely typed script, was uncovered at MIT by Mike Radzicki. All the readership will be grateful for his sharp eyesight and good fortune. The System Dynamics Society is planning eventually to make available a complete set of the MIT D-Memos on a DVD.*

*Brian C Dangerfield*
*Executive Editor*

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ARCHIVES PAPER

# Dynamic models of economic systems and industrial organizations

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*Abstract*

In the material that follows, I comment on model building from the viewpoint of its effectiveness in predicting economic behavior and its value as a guide to the management of industrial organizations and the establishment of operating policies and organizational structure. I am referring throughout to models describing sequences of events progressing in time and not mathematical models for describing a static set of relationships.

Although the text is written in a somewhat critical manner, I look on most of the statements as propositions for further consideration and discussion rather than as facts which I accept at face value. While my general propositions suggest that current economic model design techniques are not progressing in the most fruitful direction, it is certainly true that those avenues needed to be explored, that most of the efforts represent some progress, and that they provide a foundation from which we can hope to build a better system of economic and industrial firm models using techniques and machines which have become available in the last decade. Copyright © 2003 John Wiley & Sons, Ltd.

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## Part I: Model building thus far

During the last three months I have read a small part of the literature on economic models and models of the industrial firm, considering it from the viewpoint of one who is new to the field. It would be premature to have yet a definite judgment of the past approaches to model building and the techniques which are used. However, certain first impressions have been formed and these might serve a useful purpose in encouraging a reconsideration of methods which may have become accepted from custom or precedent. I make no pretense at having read all the literature of the subject, or even the best. These reservations are intended to temper what may otherwise seem an excessive forcefulness and certainty in the following material.

The literature of economics contains much material on mathematical models used to describe economic behavior. By contrast, the literature on theory of the individual industrial firm seems to contain relatively less work on mathematical models for the firm as a whole. Especially is this true for the dynamic, time-dependent interplay of money, materials, and information flow between parts of the industrial organization.

The verdict seems generally accepted that model formulation has thus far been rather unsuccessful. Many reasons are advanced for failures, including

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influences of external effects, inherent unpredictability of people, inability to get good data, impossibility of controlled experiments and many others.

A brief, and perhaps too casual, examination of past attempts at model building leads me to believe that the reasons for their limited success lie much deeper and that the difficulties are of a more fundamental nature. Some of these deficiencies are discussed in the following sections.

### *1. Closed loop systems*

One of the striking shortcomings of most economic models is their failure to reflect adequately the structural form of the regenerative loops that make up our economic system. The flows of money, materials, and information feed one another around closed re-entering paths. In one of these paths a disturbance causes a cascade of events that return to affect the initial disturbance. For example, such a system often has characteristics which can convert a single isolated impulse of disturbance into a series of oscillations.

The behavior of such loops (their tendency to amplify or dampen disturbances, their natural frequency of oscillation, their ability to shift the phase or timing of events which feed into them) is determined by characteristics which are usually omitted from the models in the literature. These characteristics include:

- The resistance to change (momentum, habit, inertia, prejudices, traditions, production lead time, etc.)
- Accumulation, particularly changes in accumulation (capacitance or reservoir effects, fluctuating inventories, changes in non-invested savings, changes in liquid assets, bank balances, depreciation reserves, etc.)
- Delays (time lag via accounting departments between actual events and resultant decisions, mail delays, freight shipment time, time perhaps measured in years for the general public to become aware of social or economic changes, times measured in weeks or months for corporate managements to conclude that new trends are not meaningless fluctuations, etc.)
- Quantizing, which makes information available on a weekly, monthly or annual basis and converts many essentially continuous activities to intermittent flows.
- Policy and decision-making criteria (rules for inventory management, dividends related to earnings and other factors, objectives of monetary policy, structure of corporate organizations). There have a first-order effect on amplification characteristics of the system.

### *2. Interrelationship of goods, money, information, and labor*

Present models neglect to interrelate adequately the flows of goods, money, information, and labor. The kinds of loops discussed in the preceding section must usually contain paths for at least two of the four quantities. A certain amount of interplay between goods and money appears in existing economic models and this interplay is fundamental to economic theory. I have seen little consideration of information flow in models but, in many situations, the information paths can be more important than the money flow. Nowhere have I seen a model which simultaneously includes the flows and mobility of all four of the above factors.

*3. Volatility and irrationality*

Over-estimation of the volatility and irrationality of the systems under study may often excuse failures that arise from models with the wrong structure. I believe that many of the characteristics of a proper model of the national economy depend on deeply ingrained mental attitudes, which may change with time constants no shorter than one or two generations of the population. Other trends like the present one toward "automation" can be properly incorporated only by including a concept like a reservoir of research results which has been filled by 20 years of steadily increasing physical research and development. The motivation to draw from this research reservoir for the purposes of "automation" is in turn traceable to 15 years of *continuous* full employment and increasing cost of labor. As an example, I suggest that a severe two-year economic recession since World War II would have renewed fears of recurring unemployment and would have established social and labor union pressures which would have set "automation" back more than ten years.

*4. Evaluation of models*

Criteria for measuring the worth of a model are not generally agreed upon. Many discussions of models contain no hint that the author considers their evaluation as pertinent, possible, or necessary. When an evaluation is made, self consistency and ability to reproduce the identical curves from which the coefficients of the model were derived is sometimes presented to demonstrate model validity.

*5. Linear equations*

Linear equations have usually been used to describe a system whose essential characteristic, I believe, arise from its non-linearities. If so, the models do not represent the influences that control economic behavior or the dynamics of industrial organizations. As a simple example, a linear model is most unlikely to exhibit sustained oscillation or fluctuations without dying out or building up continuously in amplitude.

*6. Superposition*

Models, suitable only for long-range prediction, are often used with short-term influences and fluctuations omitted. This is justifiable only if the system is sufficiently linear to permit superposition, an assumption which has not been justified or defended and which is probable untrue. Therefore, the long-range trends are probably very much a function of the short-range behavior of a system.

*7. Simultaneous algebraic equations*

Many models are formulated in terms of systems of simultaneous algebraic equations. These impress me as particularly unsuited to the kind of behavior being studied. Solving a set of simultaneous equations is equivalent to assuming a form of "market transparency". This balancing of the system at each instant in time does not seem to represent the actual behavior of social systems where a disturbance is propagated along channels that may cause repercussions for years before being dispersed throughout the system and dying out. (I am not yet satisfied that the same result cannot be achieved by properly formulated algebraic equations.)

*8. Time intervals*

Time intervals of model solutions are often too widely spaced for the predictions being attempted. For example, a model solved annually to arrive at new annual values of economic variables would, if anything, be useful in predicting future trends over a five-year period but not year-to-year variations. As a rough rule-of-thumb, one would want solutions spaced closely enough to define a smooth curve through the fluctuations in which we are interested.

*9. Differences of large magnitudes*

Total magnitudes of large quantities in the system are often used rather than developing a model formulation to favor the forces which cause changes and the resulting incremental responses. Subtracting two large inaccurate quantities from one another is a notoriously poor way to obtain an accurate difference.

*10. Model complexity*

Most models have been scaled to the builder's mathematical ability or to the limited labor available for manipulating the numerical data and solving the equations. Often such models do not reflect even what the builder believes to be the true circumstances.

*11. Explicit solutions*

A preference often is apparent for explicit mathematical solutions instead of plotted curves which fit only particular situations ("elegance" vs. brute force). Mathematical theory is not far enough advanced to give explicit solutions to problems of the complexity we face. In fact, one has difficulty imagining the existence of such compact (be all and end all) solutions to the kind of non-linear models that are needed.

*12. Symbolism*

The models and their symbolism and notation are often abstracted so far from physical and social reality that they lull one into overlooking otherwise evident departures from the world of men, money, and materials.

*13. Coefficient accuracy*

Preoccupation with the determination of numerical coefficients to two or three decimal places sometimes overshadows the meaning and significance of those coefficients, while the very structure of the model may be in doubt.

*14. Multiple regression analysis*

Multiple regression analysis is often used to obtain coefficients for equations defining economic behavior. Many of the applications of this technique seem unwarranted. When a functional relationship between a number of variables is assumed, and multiple regression analysis is used to derive the resulting coefficients, the process seems little more than an advanced form of curve fitting. Given enough degrees of freedom in the assumed equations, the fit can be arbitrarily good and the equations will reproduce the original time-series data. The value as a predictive mechanism rests, however, on the functional relationship which was assumed at the beginning. I fear that success in performing the curve-fitting operation is often assumed to justify the functional relationship. The true system may, however, be responding to entirely different inherent relationships between the variables.

The general shapes of most economic time series are sufficiently similar that one should not be surprised that good curve fitting can be achieved with incorrect functional relationships. Furthermore, some of the assumed functional relationships may be correct for a particular moment in history but be changing with time. The longer the time period over which the regression analysis is carried out, the longer the period over which the data has been smoothed and "averaged". Thus, the longer the period of the sample, the more out-of-date the results will be when applied to predicting beyond the end of the time period used for the derivation of coefficients.

*15. Assumptions*

Very often the model and its results are judged by the logic with which the model is developed out of its founding assumptions, whereas the failures probably lie in those assumptions.

*16. Omissions*

Factors and influences are often omitted because they cannot be measured accurately. Factors that are admittedly present and important are omitted rather than being included with estimates of their values. This is equivalent to estimating a zero value, which is often not the most probable value.

## Part II: A future approach to model building

Thus far, the comments have been mostly non-constructive criticism of past efforts. However, if these judgements prove to be partly correct, they may point the way toward new approaches to mathematical models which are expected to predict and describe the behavior of economic systems and industrial companies. I continue to combine the discussion of models for the economy and for the individual firm because most of the comments should be applicable to either.

In the last two decades several new techniques have been developed which do not seem yet to have had significant application to the study of economics and firms:

- The development of simple servomechanisms and their extension into complex military weapons systems has given an appreciation of the importance of inertia, elasticity, storage, and delays in determining the stability and dynamic behavior of a complex system.
- Differential equations have become widely used in engineering to describe systems in which time is the independent variable.
- The use of differential equations and their application to closed-cycle systems has been extended to the theory of sampled data systems and to consideration of the inter-relation between sampling accuracy and sampling frequency.
- The electronic digital computer has been developed to a point where routine processing of numbers costs one thousandth of what is done by hand.
- The art of simulation, wherein an analog of a real system is set up and operated at an accelerated time scale, has been well developed. System simulation has thus far most often been applied to studies of military equipment. At first, simulation was in terms of analog-type computing elements and these techniques are still being applied to the study of simple, linear systems. Of great importance to our present subject are the more recent applications of high-speed electronic digital computers to the simulation of complex, non-linear systems.

These recent techniques show promise of a new avenue of attack for understanding the firm and the economy. By coupling these with the conventional tools for the study of such systems, there is hope for an approach which can avoid the difficulties discussed in Part I.

I am very certain that the models that now become possible will be effective and of great importance in understanding and managing the individual industrial firm. With respect to the national economy as a whole, I expect the models that can be constructed in the next five years to be many times better than those of the past. Whether "many times better" is enough to bring economic models to a useful and effective capability has yet to be determined.

Comments follow on various aspects of model construction. The sections parallel those in Part I.

*1. Dynamic structure*

The greatest possible detailed attention should be given to the actual sequences of actions which take place in the system being studied and to the forces which trigger or temper such actions.

As an example of this detail, consumers do not purchase (as implied by most past models) from producers of goods. They buy from retailers, who buy from wholesale distributors, who draw goods from factory warehouses which are stocked by factory production. Such a distinction and stress on reality can have a first-order effect on the degree to which a model matches the real system in performance.

Time delays and their causes are of paramount importance. By time delay, I do not mean the occasional time shift to get a "lagged" variable for correlation purposes as used in many past models. I mean an effect which is an identifiable, organic delay in one of the flow channels of the model. Examples might be:

- the delay between actual transactions and accounting reports to corporate management;
- time necessary for the processing of orders for goods;
- time required for the processing of orders for goods;
- factory lead time between a decision to produce and actual output;
- the length of the growing season between decision to plant and actual harvesting of crops;
- the time between the collection of taxes and disbursement of government funds;
- the length of time to build new factories.

The preceding time lags tend to be of the "transportation" or "pipeline" type, having the character of a fixed delay. Something is put in at one point and makes its appearance elsewhere at a definite time later.

Another class of time delays are essentially reservoir effects (when the outlet of the reservoir is opened the effect of lowering the water level begins immediately but the result becomes progressively more apparent with the passage of time. For example:

- stock re-ordering procedures which are based on total inventory level;
- the lag between demand changes and price changes depending on depletion of inventory and re-evaluation of pricing policies;
- the progressive build up of livestock herds by withholding young stock from market to build up breeding herds;
- the progressive changes in the attitude toward depressions as the memories of the 1930's fade and the fraction of the population grows which has known only prosperity;
- the interval between an innovation being considered a luxury and being accepted as a "necessity".

Under some circumstances, time delays of a day will be significant in dynamic models of the industrial firm with other significant time constants stretching into several years (depletion of natural resources, retraining personnel and reorganization for more automatic manufacturing and data handling, depreciation of equipment, etc.).

An effective model of the national economy will certainly need to recognize delays and time constants of a month, perhaps as short as a week, and stretching in a continuous progression from there up to twenty years or more (learning time of the population; time for an obsolete, displaced and unprofitable industry to die; mobility of people from farms, coal mines and other declining industries to other occupations; time for new research results to pass through development, pilot plant, public trial, and finally become an important factor in the economy; time for new social trends to become accepted, etc.).

Several delays which are cascaded in a chain usually cannot be lumped into one overall delay, especially if inputs and outputs occur at the intermediate points.

For example, four cascaded stages of delay of three months each may produce a dramatically different result from a single stage delay with a one year time constant.

As another example, in economic system models, one should be alert to the individual time lags in a chain like the following, which might be part of the system responding to movement from a period of stable consumer production to a new higher stable level requiring new investment in production equipment:

- increase in consumption;
- depletion in retail inventory;
- depletion in wholesale inventory;
- depletion in factory inventory;
- higher level of factory production;
- continued demand for more goods;

- decision to invest in more plant;
- orders for capital equipment;
- attraction of labor to industries making capital equipment;
- lead time for producing capital equipment;
- if the capital equipment industries do not have sufficient capacity, add:
  - installation of equipment in capital goods industries;
  - lead time for producing capital goods far sale;
- installation of new capital equipment;
- factory lead time to produce consumer goods with new equipment;
- distribution of additional consumer goods;
- goods available at consumer level.

The preceding chain contains both material and information flow steps and one labor mobility item. It does not contain money flow paths, which would act as coupling channels between various parts of the chain. This chain is not a model in itself but one of the threads that might be traversed through the interconnected systems of a full model. There are actually many important sub-loops in the chain which are not indicated. When the preceding chain is detailed to include typical management policies these will introduce amplification into some steps of the sequence.

An appreciation of the controlling influence of these types of lags and delays will lead to the need to make behavior measurements which have not thus far been attempted.

As another example, one might be developing that part of an industrial firm model for the relationship between production and advertising. It will be of the utmost significance to know whether an advertising campaign is triggered and motivated by inventories of unsold goods or is tied into the company planning in a more integrated way. Further, the characteristics of the response to an advertising campaign could have a first-order effect on inventory levels, continuity of employment and profits. The presence or absence and approximate values of each of the following quantities (shown in the sketch—Figure 1) and their relationship to other factors in the company structure (such as production lead time, policies for establishing production quantities, inventory levels and policies) can make important differences in sales patterns, manufacturing flow, and money demands:

- time lag between motivating events and a decision to advertise;
- time lag from decision to appearance of the campaign;
- delay in sales build up behind initiation of campaign;
- differences (if any) between peak and steady state level of sales which continue during the campaign. (To the extent that a peak represents sales that are precipitated earlier than they would otherwise be made, they may lead to a more rapid fall off at the end of the campaign or even to a dip below the ultimate enduring level. Such a response could couple with inventory

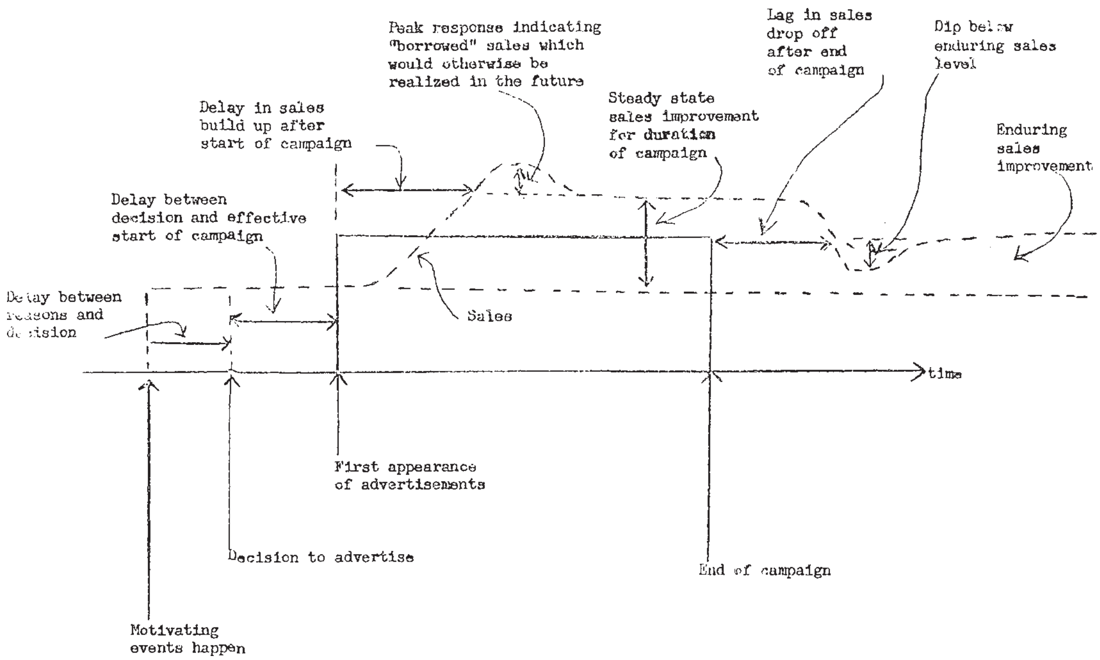


Fig. 1. Timing and extent of effects of an advertising compaign

management policies in a way to cause distressing results in production fluctuations.);

- sales improvement resulting for the duration of the advertising campaign;
- time lag and shape of sales drop off after the campaign ends;
- "permanent" sales improvement resulting from the campaign.

Estimates of the preceding quantities coupled in a single dynamic model with corresponding factors from production, inventory, financial flow, labor (availability, normal attrition) and sales organizational structure, would go far in guiding decisions on intermittent vs. continuous advertising, duration of campaigns, importance of special promotional efforts, etc.

*2. Information flow and decision criteria*

A model which does not take explicit recognition of information flow channels and information transformation with time and transmission can hardly expect to reflect the real economic world. A proper model will not be constructed on external symptoms of the economy but on the basic processes going on beneath the surface. These basic processes will include the handling and manufacturing steps for goods, group and mass population reaction times, and characteristics of our communications methods such as:

- education;
- response to advertising;
- readjustment of population and working force distribution to new economic circumstances;
- time lags in discovering "greener pastures on the other side of the fence";
- jockeying between labor organizations year-by-year to equal or better each other's wage scales;
- shifts between consumer and capital goods industries.

Decision criteria will need to be re-examined for the nature of the coefficients used. We must depart from the current practice of assuming that current decisions in the economy depend on only the current values of gross economic variables (or some of these lagged by a year). The extended past history of many of the economic variables condition people to certain reactions, also the yearly changes in a variable (liquid asset *increases* or *decreases*, wage *changes*, price *changes*, *new* products) can be more important than the total magnitudes of these quantities. For example, in many situations, *rate of decrease* in unused (excess) plant capacity might be a much more powerful factor in determining new investment plans than total production, total profits, or total business liquid assets.

The proper decision criteria will probably often be found to be unsymmetrical with respect to the direction of change of economic variables. (Is not a reduction in standard of living a more powerful influence than the availability of a higher standard? The fall in corporate dividends will *lag* the fall in earnings during a steady business decline, but may *lead* the early part of the ensuing economic re-activation. Such lack of symmetry can be traced to the motivations, hopes, objectives and optimism of the people involved.)

*3. Continuity and momentum of the system*

When an economic model is built with the proper structure, we may find that much of the apparent volatility and irrationality now ascribed to human behavior will disappear. The enduring fundamental characteristics which I have been discussing are certainly fully capable of coupling in ways which will cause the kinds of fluctuations that one observes in the real economic world. There is no necessary contradiction between the concept of stable controlling principles and the kind of variability that the surface of the system exhibits. Furthermore, the enduring principles will probably be found only one level deeper in the social structure than the level which is now being searched by economists and model builders. I anticipate no great difficulty in identifying those principles which we seek once the search is concentrated in the proper area. (Look not in the best light but where the object was lost.) Many of the answers we want will come from contemplation of individual, group, and mass behavior.

Perhaps here is the place to comment on economic model building versus the business man's intuitive judgments. One would be hard pressed to judge which has been the more unsuccessful in predicting economic conditions. May there not be a reason for this equality? I suggest that economic models are often an orderly arrangement of superficial manifestations while the business man's intuition represents a disordered accumulation of basic insights into how people and social systems react. The hope for the future lies in generating an orderly arrangement of basic insights.

### *4. Model effectiveness*

I suggest that a model should be judged principally by its ability to predict behavior in regions whose data were not used in constructing the model. A model can be valuable in "explaining" past actions, but there is great danger in the explanatory model that the model building process becomes only one of curve fitting and not a real determination of cause-and-effect relationships. A model that can predict will be the one most persuasive as an explanation.

### *5. Non-linear systems*

Almost every characteristic that one examines in the economic system is highly non-linear. A non-linear system, operating most of the time at the limit of some of its parameters, can hardly be expected to behave like an idealized linear model. The only solution that I see is to fortify our courage and dive into an intricate situation. Non-linear systems can be handled for particular initial conditions. The results may not be directly useful under other conditions, but there is no evidence that the systems we find of interest here possess general, explicit solutions. Indeed, the world of the engineer, which has long been viewed by the social scientist as relatively linear and well behaved, is rapidly reaching a degree of complexity where the problems are becoming comparable to those in economics. Help from physical sciences can therefore be expected in building a body of knowledge for handling such non-linear systems involving both information and material flows.

### *6. Long- and short-range responses*

For systems which are not linear, limitation of a model to influences presumably affecting only medium—long-range trends (3 to 30 years) should be viewed with skepticism. Short term (1 month to 5 years) behavior might be studied in the absence of long-term trends but not vice versa. In a sufficiently non-linear system we should expect, until shown otherwise, that the short-term behavior may set the stage for the longer-term reactions.

*7. Differential equations*

The behavior that we wish to describe in economic systems seems much better described by non-linear differential equations than by the algebraic equations and matrix operations which are so often used. Delays, momentum, elasticity, reservoirs, and accelerations are the fundamental quantities which differential equations have been developed to describe. They are the quantities which we wish to use for our underlying principles of the economic world. The equations will have non-linear coefficients, complexly inter-related variables, "pipe-line" delays, variables which can be only positive, etc. In practice, they will need to be handled as incremental difference equations so that numerical solutions can be obtained.

*8. Incremental time intervals.*

The incremental time intervals for which the variables of a model are solved step-by-step in time must be much shorter than often supposed. For most industrial company models this interval will be about a week, probably as short as daily in many studies. For models of the national economy as a whole it is unlikely that the time interval can be longer than one month and it is entirely possible that weekly intervals might be necessary.

This solution interval is unrelated to the interval at which national statistics and economic indicators are measured. The model should generate the instantaneous values of the variables which exist in the real system, whether or not these can be measured. The measurability of some of these variables is immaterial to the structure of the model and the incremental time steps through which it is advanced. The frequency of collection of statistics will only determine the frequency with which the model can be compared with reality, and in turn will affect the ease of getting the model structure and coefficients to converge toward their real counterparts.

The incremental time interval used for model solution will, on the other hand, be related to the lengths of the time constants which have been incorporated in the structure of the model. As a rough generalization, it will be necessary to have several solution intervals within the shortest time delay which is recognized in the model. Correspondingly, the model will need several solution intervals in a half-cycle of the highest frequency response which is to be generated at any point in the model.

*9. Incremental changes in variables*

Often a choice will be available between (1) formulating a model in terms of the motivations that cause incremental changes in a variable and, in contrast, (2) a formulation in terms of equations which generate a balance between the full magnitudes of the variables. By choosing the first alternative, the new value of a variable can be found by solving the equations for its incremental change and then adding the change to the preceding full value of the variable.

### *10. Model complexity*

By following the procedures being suggested, we will be led to models of much greater complexity and completeness than have thus far been used. Several hundred variables may be expected in any reasonably adequate model of the aggregate economy.

This is not as frightening as it may sound. The labor involved in the step-by-step solution of a set of differential equations for one single solution is much less than that of solving a set of algebraic equations in the corresponding number of variables. For a properly formulated set of equations, I have no great concern about round-off errors or other idiosyncrasies introduced by the mathematical technique itself. The complexity of such a model is comparable to some that have already been handled in weapon system studies. The modern high-speed digital computer has an adequate capacity for doing the calculating.

### *11. Empirical solutions*

With the kind of model being discussed, an explicit solution will not be possible. Solutions (predictions of ensuing response) can be made for various assumptions about the model structure and initial values of the variables. The sensitivity of model behavior to changes in constants, individual values of variables, or model structure can be determined by trial. General relationships between variables can be determined in the vicinity of a particular operating point but these will not necessarily hold in other regions of model operation unless verified by trial in those regions.

Any generalizations which become possible develop out of a study of particular situations. At times the non-linear model may demonstrate that a linearized approximation is indeed valid for some restricted purpose.

### *12. Symbolism*

The proposed model structure and method of solution retain a one-to-one correspondence between the presumed form of the real economic world and the quantities, coefficients, variables, and decision criteria of the model. Formulation in terms of a "flow diagram" is possible so that a pictorial representation of the relationships within the system is available at all times. The solution with a digital computer is a "simulation" process in which the information, money, goods, and people are moved time-step-by-time-step from place to place. Numerical values retain their identity with their real counterparts, and changes with time in any or all parts of the system can be traced.

One can presume that errors in the assumptions underlying the model will be more evident if the correspondence between the individual units of the model can be continuously related to their real sources.

*13. Coefficient accuracy*

A new attitude toward the determination of numerical coefficients may be demanded. At some points one may have to choose between a structure one believes is correct and coefficients which can be measured with accuracy. As I now view it, I would prefer a structure in which I had confidence using intuitively estimated coefficients rather than an unlikely structure and functional relationship for which the coefficients could be derived accurately from statistical data. The necessity for making this choice is probably not permanent because the need for new numerical data will lead to discovering methods for their measurement.

*14. Multiple regression analysis*

Multiple regression analysis should be carefully reviewed to establish the conditions under which it can be safely used. As a proposal some of these conditions are:

- when one is confident that one knows the correct functional relationship between the variables whose coefficients are being sought;
- when there is reason to believe that the coefficients and functional relationships are not changing over the time period of the sample (when time series are the source of the data);
- when the relationships are linear.

My comments about this technique are only impressions and not based on any real study of the method but there seems enough controversy over multiple regression applications to embolden me to make these observations.

The technique can probably safely be used for deriving useful relationships from a cross section of samples at one point in time. It looks particularly doubtful for studying the relationships between two variables which are part of the same closed-loop regeneration system.

## Part III

Part III will eventually contain examples, a discussion of the areas where new theory will need developing, and the magnitude of the effort required to achieve some of the results discussed herein.