

The Impact of Information Technology on the Temporal Optimization of Supply Chain Performance

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Abstract

The objective of this paper is to suggest a systematic means by which the timing and focus of information technology policies can be used to optimize supply chain performance and reduce production times. This is done by extending an earlier statistical physics model for quasi-static situations to examine temporal phenomena in manufacturing supply chains.

A simple model is considered in which the product manufacturing process in a supply chain resembles the flow of a fluid in a pipe. At each position in the pipe, value is added to the fluid to produce the finished product at the exit orifice. Following the statistical physics model, information technology policy impacts the rate of change of the flow velocity by the action of a statistical physics-derived effective force.

It is found that wave phenomena naturally occur in the inventories in supply chains. A simple quasilinear analysis of the phenomena shows that information technology policy can be used to adjust timing and position issues to allow the chain to resonate with the propagating waves, in a manner that most effectively in reduces overall production times. This is the first step in an analytical mathematical approach that allows the evaluation of information technology architectures and topologies that optimize production outputs and minimize production disruptions.

1. Introduction

Many government policies have been implemented over the years to stimulate innovation and promote entrepreneurship in the private sector. These have taken many forms. At one end of the spectrum, tax incentives have been provided to encourage investments in research and development. To encourage innovation in areas of particular value to the national interest, expedited review and issuance of patents have been instituted. Procurement policies of the various government agencies have included special set-asides for small entrepreneurial businesses. An effective small business innovation research (SBIR) program has been in place for several years to provide research and development funds to entrepreneurs. One important policy focus had been overlooked: A policy that encourages Information Technology investment in the small business community.

It was demonstrated that IT investment by small firms not only increases firm output, it also has a second positive impact of job creation (Dozier and Chang 2006). IT may play a critical role in increasing a small firms ability to survive in the dynamic environment of a manufacturing supply chain. All manufacturing firms small and large are impacted by supply chain issues. General Motors employs 1700 people just to manage their supply chain. Most small firms are tossed about on the complex and turbulent disruptions that propagate through supply chains. Sterman and others have

done much to demonstrate the destructive nature of even a single change in simple supply chain (Beer Game simulation) (Sterman 2000). While this has done a great service by increasing awareness of the problem, it does little to provide tools that can be used to optimize supply chain performance or minimize damage caused by mismanagement of inventory stores and flows.

Of special concern is a striking disparity between the long time scales of government and large firm policy decisions compared to the market-driven increasingly short time scales of the private sector (Koehler 2003). The synergy between the public and private sector policy is diminished by the differences in these time scales as well as many underlying difference in cultures.

Government and large firm policy decisions can affect many aspects of the manufacturing process that act as tsunami's of change that can overwhelm the small firm. This paper begins the effort to create a mathematical model that can be easily simulated to allow managers to minimize the negative impact of unexpected external perturbations such as government and large firm supply chain policy shifts. The ability to optimize production times in an ever shifting manufacturing supply chain is critical to small firm survival.

Section 2 describes a simple model of a manufacturing supply chain that is based on the analogy between the flow of an evolving product through a manufacturing supply chain from its basic component elements at the input end to its finished state at the output end, and the flow of a fluid through a pipe.

Section 3 demonstrates that the equations corresponding to the simple supply chain model have oscillatory wave-like solutions. These describe inventory perturbations that travel along the supply chain.

Section 4 then introduces the subject of external policy actions on the supply chain. It builds on our earlier statistical physics work that showed that information transfer could be described by a "force" that causes a change in the unit cost of production. In the model this force results in an acceleration of the flow velocity in the supply chain flow. A quasilinear equation is developed that shows how a net (secular) change can occur in the rate at which an evolving product moves through the supply chain.

Section 5 summarizes and discusses the model results and their implications for the development of information technology topologies that minimizes the disturbances lowers unit production costs

2. Supply chain model

In this Section equations will be developed for a "fluid in a pipe" model of a manufacturing supply chain. Thus, in a supply chain, the basic component elements involved in producing a product enter at the starting end, and a finished product emerges at the output end. In between, value is added at each step by combining, modifying, manipulating, assembling, etc. the results of the previous step to produce input to the next step. In some ways this can be likened to fluid flowing in a pipe, with different "colors" added to the fluid as it flows through the pipe. At the starting end of the pipe, the entering fluid would be clear, whereas at the output end a fluid with a rich blend of colors would emerge.

In this model, the manufacturing supply chain has a very simple daisy chain topology for the interconnection of the information system. This can be characterized as consisting of several stages arranged in tandem. The n th stage receives information input from the $(n-1)$ stage and delivers information output to the $(n+1)$ stage. This occurs for the n (total) stages in the chain. A conservation equation can be written for the number of units being manufactured at any particular time at any stage.

To simplify the mathematics, the following approximations will be made:

1. The number of stages is large therefore the discrete variable n ($n=1, 2 \dots, n(\text{total})$) is replaced by a continuous variable x that runs from 1 to $n(\text{total})$, and that takes on the value of n when x has an integer value. This permits the replacement of difference expressions by differential expressions.

2. The total number of stages $n(\text{total})$ will be assumed to be so large, $n(\text{total}) \gg 1$, that end effects of the chain will be ignored.

It is apparent that both of these approximations can be relaxed easily: however, the approximations will be used in this short paper in order not to detract from the essential features of the results.

After replacing the variable n with x , we can then designate the rate, at which an entity moves through the supply chain by the velocity variable v ,

$$v = dx/dt \quad [1]$$

By analogy with the dynamic equations for fluid flow, the time rate of change of the velocity v will be given by an equation patterned after the $F=MA$ equation of kinetics:

$$F = dv/dt \quad [2]$$

F is an appropriate “force” that drives the process.

What is this force? It was shown by applying the Lagrange multiplier approach of statistical physics to determine the most likely distribution of unit costs of production in a time-independent situation, that a statistical physics force can be uniquely determined from this distribution (Dozier and Chang 2005). A common feature of both situations is the effective force that

drives the phenomena, and this is derived from the partition function.

Specifically, the most likely distribution of unit costs of production gives rise to a partition function

$$Z = \sum \exp[-\beta C(i)] \quad [3]$$

where $C(i)$ is the unit cost of production of the i th company in the distribution.

Associated with this partition function is a “Helmholtz free energy” F defined by

$$\exp[-\beta F] = Z \quad [4]$$

where β – which we call the “bureaucratic factor” - plays the role of an inverse temperature, and is a measure of how much dispersion there is in the unit costs of production. The statistical physics formalism then gives for the force $f(\xi)$ associated with the variation of any parameter ξ of the system

$$f(\xi) = \partial F / \partial \xi \quad [5]$$

For our purposes, we can later choose the manufacturing system parameter ξ to be any quantity that is the focus of the intervention. For example, ξ can be the amount of technology change induced by a government incentive or a prime contractor’s new requirement.

To obtain eq. [2] from eq. [5], we assume that the change in the free energy F of the system associated with $f(\xi)$ translates directly to a change in the rate at which production processes occur in the supply chain. In that case, the force F in eq. [2] is simply proportional to $f(\xi)$.

$$F = \alpha f(\xi) \quad [6]$$

Where α is a constant of proportionality that depends on the particular system parameter ξ that is changed.

Now introduce a simple differential distribution function $f(x,v,t)dx dv$ that denotes the number of production units in the intervals dx and dv at x and v at the time t . A conservation equation can be written for f :

$$\partial f / \partial t + \partial [f dx / dt] / \partial x + \partial [f dv / dt] / \partial v = 0 \quad [7]$$

This equation simply states that the change of $f dx dv$ is due only to the divergence of the flow into $dx dv$.

Using equations [1] and [2] this can be rewritten.

$$\partial f / \partial t + \partial [fv] / \partial x + \partial [fF] / \partial v = 0 \quad [8]$$

Since x and v are independent variables,

$$\partial v / \partial x = 0 \quad [9]$$

If the force F does not depend on v ,

$$\partial F / \partial v = 0 \quad [10]$$

then eqs [8]-[10] yield.

$$\partial f / \partial t + v \partial f / \partial x + F \partial f / \partial v = 0 \quad [11]$$

This has the familiar form of the Vlasov equation for collisionless plasmas. It is expected that many of the benefits of this application form for many-body problems should also apply here (Spitzer 1956).

It is convenient to deal with is the number of production units in the interval dx and x at time t ,

$$N(x, t) = \int dv f(x, v, t) \quad [12]$$

and

$$V(x, t) = (1/N) \int v dv f(x, v, t) \quad [13]$$

where $N(x,t)dx$ and $V(x,t)$ is the average velocity of flow of the production units at x at time t .

By taking the v^0 and v^1 moments of eq. [11] – see, e.g. (Spitzer 1956), we find

$$\partial N / \partial t + \partial [NV] / \partial x = 0 \quad [14]$$

and

$$\partial V / \partial t + V \partial V / \partial x = F_1 - \partial P / \partial x \quad [15]$$

where $F_1(x,t)$ is the total force F acting per unit dx and P is a “pressure” defined by the dispersion of the velocities v about the average velocity V :

$$P(x, t) = \int dv (v-V)^2 f(x, v, t) \quad [16]$$

We can write eq. [16] in the form

$$P(x, t) = \int dv (v-V)^2 f(x, v, t) = N(x, t) (\Delta v)^2 \quad [17]$$

where

$$(\Delta v)^2 = \int dv (v-V)^2 f(x, v, t) / N(x, t) \quad [18]$$

This is a convenient form, since we it shall assume for simplicity that the velocity dispersion $(\Delta v)^2$ is independent of level x and time t . In that case, eq. [15] can be rewritten as

$$\partial V / \partial t + V \partial V / \partial x = F_1 - (\Delta v)^2 \partial N / \partial x \quad [19]$$

This implies the change in velocity flow is impacted by the primary forcing function and the interacting gradients at production locations. Equations [14] and [19] are the basic equations that we shall use in the remainder to describe temporal phenomena in our simple supply chain model.

3. Supply chain normal modes

A technique that has been found to be very useful in the analysis of temporal phenomena in physical systems is to first identify the normal modes of the system. The normal modes are naturally occurring oscillatory perturbations of the system, and they are important in determining the response of the system to external forces. If an external force has the same spatial form and frequency as a normal mode, then that mode is resonantly excited, and can have quite large amplitude.

Thus, to determine the response of a supply chain to external forces imposed by an external policy change, it should be useful to first determine the natural normal modes of the supply chain.

The normal modes are usually obtained by looking at the perturbations of the system about its steady state. Accordingly, let us introduce the expansions

$$N(x,t) = N_0 + N_1(x,t) \quad [20]$$

and

$$V(x,t) = V_0 + V_1(x,t) \quad [21]$$

about the level- and time-independent steady state density N_0 and velocity V_0 . (We can take the steady state quantities to be independent of the level in the supply chain, since we are considering long supply chains in the approximation that end effects can be neglected.)

Substitution of eqs. [20] and [21] into eqs. [14] and [19], we see that the lowest order equations (for N_0 and V_0) are automatically satisfied, and that the first order quantities satisfy

$$\partial N_1 / \partial t + V_0 \partial N_1 / \partial x + N_0 \partial V_1 / \partial x = 0 \quad [22]$$

and

$$\partial V_1 / \partial t + V_0 \partial V_1 / \partial x = F_1(x,t) - (\Delta v)^2 \partial N_1 / \partial x \quad [23]$$

where $F_1(x,t)$ is regarded as a first order quantity.

For an oscillatory disturbance,

$$N_1(x,t) = N_1(x) \exp(i\omega t) \quad [24]$$

$$V_1(x,t) = V_1(x) \exp(i\omega t) \quad [25]$$

and for a normal mode, there is no external applied for F_1 .

Since the coefficients in eqs. [22] and [23] are independent of x , the equations have eigenfunctions (normal modes) of the form

$$N_1(x) = N_I \exp(-ikx) \quad [26]$$

$$V_1(x) = V_I \exp(-ikx) \quad [27]$$

i.e. the normal modes are propagating waves:

$$N_1(x,t) = N_I \exp[i(\omega t - kx)] \quad [28]$$

$$V_1(x,t) = V_I \exp[i(\omega t - kx)] \quad [29]$$

With these forms, eqs. [22] and [23] become

$$i(\omega - kV_0)N_I + N_0 ikV_I = 0 \quad [30]$$

$$iN_0(\omega - kV_0)V_I = -ik(\Delta v)^2 N_I \quad [31]$$

In order to have nonzero values for N_I and V_I , these two equations demand that

$$(\omega - kV_0)^2 = k^2(\Delta v)^2 \quad [32]$$

Eq. [32] has two possible solutions

$$\omega_+ = k(V_0 + \Delta v) \quad [33]$$

$$\omega_- = k(V_0 - \Delta v) \quad [34]$$

The first corresponds to a propagating supply chain wave that has a propagation velocity equal to the sum of the steady state velocity V_0 plus the dispersion velocity width Δv . The second

corresponds to a slower propagation velocity equal to the difference of the steady state velocity V_0 and the dispersion velocity width Δv .

A different supply chain model was considered by Dozier and Chang (Dozier and Chang 2005). In that model (the continuum limit) is equivalent to the model discussed here when either V_0 or Δv is 0.

4. Interactions

As indicated earlier, our focus in this paper is on the effect of external interactions such as government policy and supply chain perturbations on the rate at which an evolving product moves along the supply chain. In Section 2, it was pointed out that this occurs in the equations through a force $F_1(x,t)$ that acts to accelerate the rate. From the discussion of Section 3, we expect that this force will be most effective when it has a component that coincides with the form of a normal mode, since then a resonant non destructive interaction can occur.

To see this resonance effect, it is best to present the force F in its Fourier decomposition

$$F_1(x,t) = (1/2\pi) \int \int d\omega dk F_I(\omega,k) \exp[i(\omega t - kx)] \quad [35]$$

where

$$F_I(\omega,k) = (1/2\pi) \int \int dx dt F_1(x,t) \exp[-i(\omega t - kx)] \quad [36]$$

With this Fourier decomposition, each component has the form of a propagating

wave, and it would be expected that these propagating waves are the most appropriate quantities for interacting with the normal modes of the supply chain.

Our interest is in the change that F_1 can bring to V_0 , the velocity of product flow that is independent of x . By contrast, F_1 changes V_1 directly, but each wave component causes an oscillatory change in V_1 both in time and with supply chain level, with no net (average) change.

To obtain a net change in V , we shall go to one higher order in the expansion of $V(x,t)$:

$$V(x,t) = V_0 + V_1(x,t) + V_2(x,t) \quad [37]$$

On substitution of this expression into eq. [19], we find the equation for $V_2(x,t)$ to be

$$\begin{aligned} N_0(\partial V_2/\partial t + V_0\partial V_2/\partial x) + N_1(\partial V_1/\partial t + V_0\partial V_1/\partial x) + N_0 V_1\partial V_1/\partial x = \\ - (\Delta v)^2 \partial N_2/\partial x \end{aligned} \quad [38]$$

Fourier analysis of this equation, using for the product terms, the convolution expression:

$$\begin{aligned} \iint dxdt \exp[-i(\omega t - kx)] f(x,t)g(x,t) \\ = \iint d\Omega dK f(-\Omega + \omega, -K + k)g(\Omega, K) \end{aligned} \quad [39]$$

where

$$f(\Omega, K) = \iint dxdt \exp[-i(\Omega t - Kx)] f(x,t) \quad [40]$$

$$g(\Omega, K) = \iint dxdt \exp[-i(\Omega t - Kx)] g(x,t) \quad [41]$$

Since we are interested in the net changes in V_2 – i.e. in the changes brought about by F_1 that do not oscillate to give a zero average, we need only look at the expression for the time rate of change of the $\omega=0, k=0$ component, $V_2(\omega=0, k=0)$.

From eq. [38], we see that the equation for $\partial V_2(\omega=0, k=0)/\partial t$ requires knowing N_I and V_I . When $F_I(\omega, k)$ is present, then eqs. [30] and [31] for the normal modes are replaced by

$$i(\omega - kV_0)N_I(\omega, k) + N_0 ikV_I(\omega, k) = 0 \quad [42]$$

$$\begin{aligned} i N_0 (\omega - kV_0)V_I(\omega, k) = \\ -ik (\Delta v)^2 N_I(\omega, k) + F_I(\omega, k) \end{aligned} \quad [43]$$

These have the solutions

$$\begin{aligned} N_I(\omega, k) = \\ -ik F_I(\omega, k) [(\omega - kV_0)^2 - k^2 (\Delta v)^2]^{-1} \end{aligned} \quad [44]$$

$$V_I(\omega, k) = -i \{F_I(\omega, k)/N_0\} (\omega - kV_0) [(\omega - kV_0)^2 - k^2 (\Delta v)^2]^{-1} \quad [45]$$

Substitution of these expressions into the $\omega=0, k=0$ component of the Fourier transform of eq. [38] gives directly

$$\begin{aligned} \partial V_2(0,0)/\partial t = \iint d\omega dk (ik/N_0^2) (\omega - kV_0)^2 [(\omega - kV_0)^2 - k^2 (\Delta v)^2]^{-2} F_I(-\omega, k) \\ F_I(-\omega, k) \end{aligned} \quad [46]$$

This resembles the quasilinear equation that is has been long used in plasma physics to describe the evolution of a background distribution of electrons subjected to Landau acceleration (Drummond and P. 1962).

As anticipated, a resonance occurs at the normal mode frequencies of the supply chain, i.e. when

$$(\omega - kV_0)^2 - k^2 (\Delta v)^2 = 0 \quad [47]$$

First consider the integral over ω from $\omega = -\infty$ to $\omega = \infty$. The integration is uneventful except in the vicinity of the resonance condition where the integrand has a singularity.

To determine the effect of the singularity – which as we have seen lies on the real ω axis, circumscribe a very small circle about the singularity in the complex- ω plane. Perform a contour integration of the integrand around the circle in a counterclockwise direction. When the circle is very small, the contribution of the top half of the circle is the same as that from the bottom half. We can then use the familiar result from the theory of analytic functions

$$\int dz f(z)/(z-z_0)^{n+1} = 2\pi i f^{(n)}(z_0)/n! \quad [48]$$

to evaluate the contribution of the singularity. For the rationale for this procedure, see. e.g., (Chang 1964).

When this is applied to eq. [46], we find that when the bulk of the spectrum of $F_1(x,t)$ is distant from the singularities, the principal part of the integral is approximately zero, where the principal part is the portion of the integral when ω

is not close to the singularities at $\omega = k(V_0 \pm \Delta v)$. This leaves only the singularities that contribute to $\partial V_2(0,0)/\partial t$.

The result is the simple expression:

$$\begin{aligned} \partial V_2(0,0)/\partial t = \pi/(N_0^2 \Delta v) \int dk (1/k) \\ [F_I(-k(V_0 - \Delta v), -k) F_I(k(V_0 - \Delta v), k) - \\ (-k(V_0 + \Delta v), -k) F_I(k(V_0 + \Delta v), k)] \quad [49] \end{aligned}$$

Equation [49] suggests that any net change in the rate of production in the entire supply chain is due to the Fourier components of the effective statistical physics force describing the external interactions with the supply chain, that resonate with the normal modes of the supply chain.

5. Discussion

The purpose of this paper has been to begin exploration of the effects of external interactions on the production rate in a manufacturing supply chain. Of special interest is impact of a simple “daisy chain” topology of the information systems that connects the supply chain as it pertains to the timing of the interventions that should have the optimum positive effect.

To study these two questions, a simple model of the supply chain has been considered in which the evolving product moves through the supply chain in somewhat the same way that fluid moves through a pipe. At each stage of the supply chain, value is added, until at the output end of the chain the final finished product emerges.

The fluid-like equations show that naturally occurring oscillatory phenomena occur in the supply chain. These oscillations take the form of propagating waves. For the simple model treated in this paper, the waves propagate at two velocities that depend on the average steady state rate of production and the dispersion of rates about this average.

The most interesting finding of the analysis is that net changes in the production rate of the entire chain are related to the Fourier components of the intervention that resonate with the propagating waves of the supply chain. This is very similar to physical phenomena in which an effective way to cause growth of a system parameter is to apply an external force that is in resonant with the normal modes of the system.

Although the linear supply chain model treated here is quite simplistic, we believe that for more complex manufacturing clusters, the same type of result would occur. We see that the simple linear topology does nothing to dampen the effects of interventions at various locations in the supply chain. The model does provide enough information to allow examination of the optimal timing for the interventions: something not possible in the simulation models to date. The most effective interventions will be those that have time scales comparable to the natural time scales of the systems, and that are applied to the systems so as to mimic the naturally occurring patterns of the normal modes of the systems. Future work will examine the impact of various IT topologies and the ability of shared knowledge between co suppliers to

optimize their cooperative production rates. It will also be interesting to determine the minimal amount of information that is required for positive cooperation.

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